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Engineering Notes

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Transfer Matrix Vibration Analysis of Cable-Supported Hoop Platforms: Additional Results

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Nomenclature

$[C]$ = cable force matrix
k_{c_i} = cable stiffness
l_{c_i} = cable length
p_{c_i} = cable preload
β_i = angle between cable and normal to hoop plane
δ_i = cable offset angle
γ = phase angle
ω = natural frequency

Introduction

KNOWLEDGE of the free-vibration characteristics of large space structures is usually considered prerequisite to insuring satisfactory operation. Polygonal platforms, supported and stiffened with cables, are often used as major components for such structures. Hexagonal platforms have been analyzed for their free-vibration characteristics using a NASTRAN finite element program[1] and a transfer matrix method exploiting cyclic symmetry.[2] The transfer matrix method taking advantage of the periodicity of the structure was shown to be an efficient alternative, based on a CPU time and required storage comparison.

Analysis

The hexagonal hoop platform in question is shown schematically in Fig. 1. Its members consist of rectangular aluminum beams 6.35 mm thick and 50.8 mm wide. They are interconnected rigidly at their ends, and their length is such that a hoop radius measured from the platform center to a joint between members is 1 m. The cable mass is assumed negligible, and neither masses offset from the hoop members' principal axes nor transverse shear deflections are accounted for. Details of the notation and methods are given in Ref. 2. The only way this analysis differs from that of Ref. 2 is that the cable force matrix reflects cables oriented out of the hoop plane and a sum of cable effects rather than just one cable at each vertex. Hence, nonzero elements in the cable force matrix $[C]$ become

$$a_{1,4} = -\sum_{i=1}^{2}\left[\sin^2\beta_i\cos^2\delta_i\left(k_{c_i}-\frac{p_{c_i}}{l_{c_i}}\right)+\frac{p_{c_i}}{l_{c_i}}\right]$$

$$a_{1,8} = \sum_{i=1}^{2}\sin\beta_i\cos\beta_i\cos\delta_i\left(k_{c_i}-\frac{p_{c_i}}{l_{c_i}}\right)$$

$$a_{1,10} = \sum_{i=1}^{2}\sin\delta_i\cos\delta_i\sin^2\beta_i\left(k_{c_i}-\frac{p_{c_i}}{l_{c_i}}\right)$$

$$a_{5,4} = a_{1,8}$$

$$a_{5,8} = -\sum_{i=1}^{2}\left[\cos^2\beta_i\left(k_{c_i}-\frac{p_{c_i}}{l_{c_i}}\right)+\frac{p_{c_i}}{l_{c_i}}\right]$$

$$a_{5,10} = -\sum_{i=1}^{2}\sin\beta_i\cos\beta_i\sin\delta_i\left(k_{c_i}-\frac{p_{c_i}}{l_{c_i}}\right)$$

$$a_{9,4} = \sum_{i=1}^{2}\sin\delta_i\cos\delta_i\sin^2\beta_i\left(k_{c_i}-\frac{p_{c_i}}{l_{c_i}}\right)$$

$$a_{9,8} = a_{5,10}$$

$$a_{9,10} = -\sum_{i=1}^{2}\left[\sin^2\beta_i\sin^2\delta_i\left(k_{c_i}-\frac{p_{c_i}}{l_{c_i}}\right)+\frac{p_{c_i}}{l_{c_i}}\right]$$

Results and Discussion

When cable stiffness and geometry are identical above and below the hoop, the terms in the cable force matrix $[C]$ that couple in-plane and out-of-plane motion of the structure vanish. Thus, the resulting modes are uncoupled as regards in-plane and out-of-plane motion.

Table 1 lists natural frequencies for various cable and cyclic symmetry phase angles[3] γ for $k_c = 1.693 \times 10^5$ N/m. Figures 2 and 3 show the first three in-plane and out-of-plane mode shapes, respectively, for the cable/hoop configuration with $\beta_1 = 45$ deg and $\beta_2 = 135$ deg. Heavy lines show members in the position of maximum deflections, and dotted lines show their undeflected positions. Light, solid lines either connect corresponding undeflected and deflected points or show cable positions with the hoop deflected to the full (normalized) vibration amplitude. As in Ref. 2, some natural frequencies have two modes, for the same cyclic symmetry phase angle γ, which are orthogonal to each other.

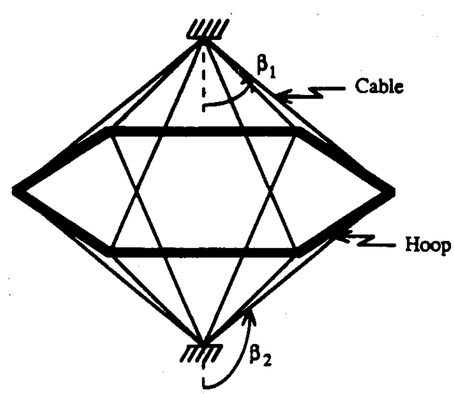

Fig. 1 Cable-supported hexagonal hoop with cable angle.

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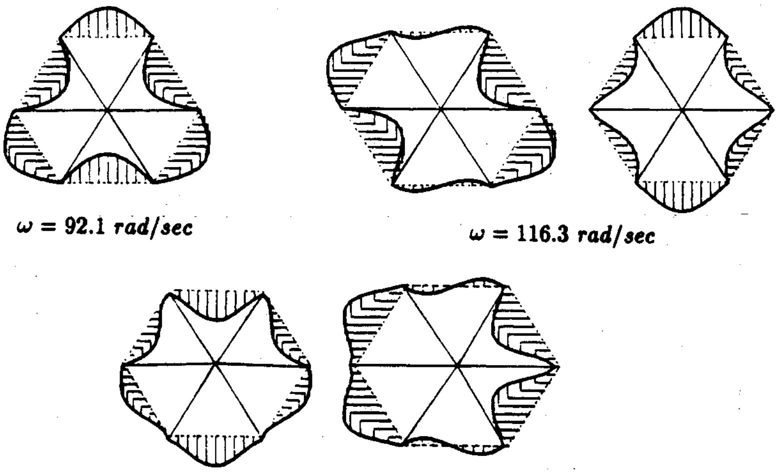

Fig. 2 In-plane mode shapes.

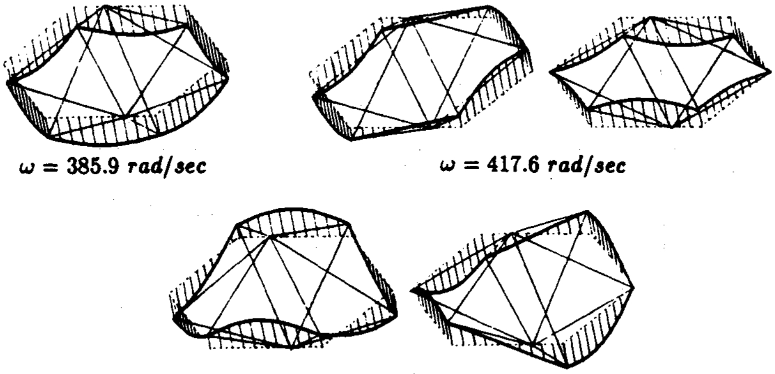

Fig. 3 Out-of-plane mode shapes.

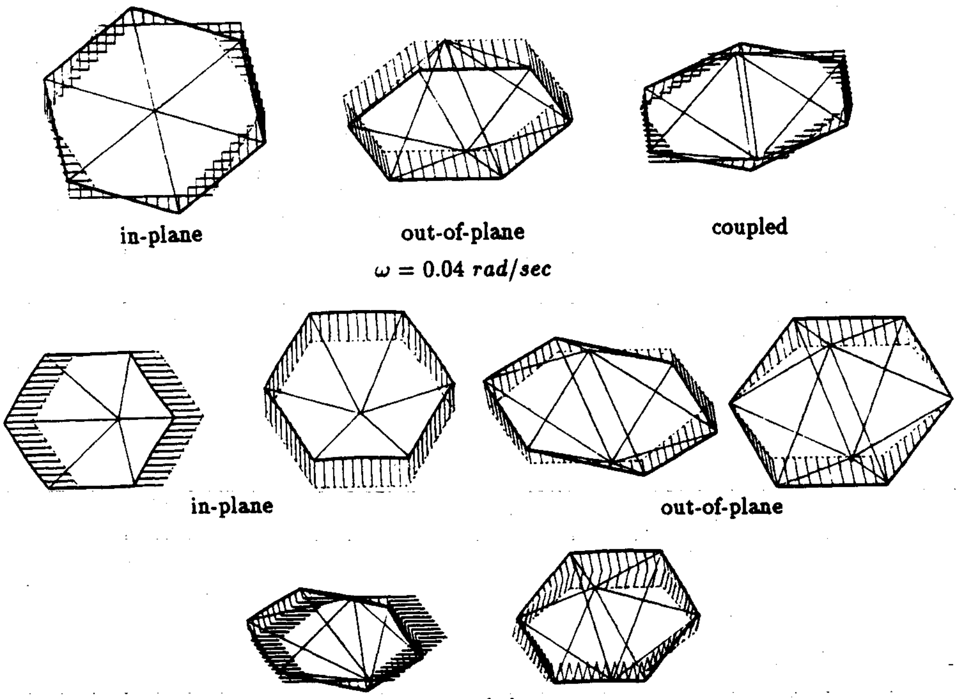

Fig. 4 Coupled rigid body mode shapes.

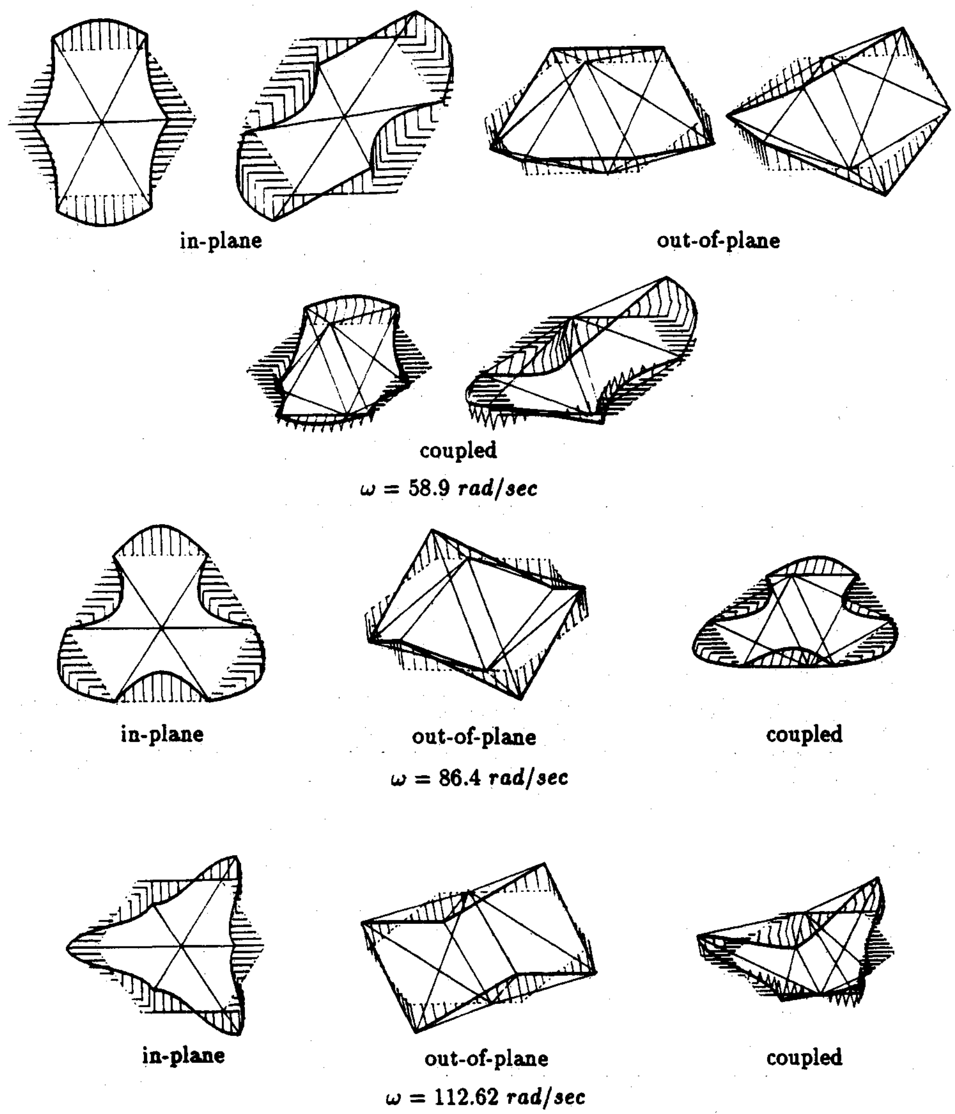

Fig. 5 Coupled flexible mode shapes.

Table 1 Uncoupled natural frequencies (rad/s)[a]

β, deg	γ	In-plane	Out-of-plane
$\beta_1 = 45$	0	209.4	385.9
$\beta_2 = 135$	$\pi/3$	162.5	417.6
	$2\pi/3$	116.3	522.2
	π	92.1	739.5
$\beta_1 = 60$	0	209.5	290.6
$\beta_2 = 120$	$\pi/3$	166.1	316.3
	$2\pi/3$	116.9	404.5
	π	92.1	541.3
$\beta_1 = 75$	0	209.4	158.3
$\beta_2 = 105$	$\pi/3$	167.5	172.7
	$2\pi/3$	117.1	227.4
	π	92.1	302.8

[a]$k_c = 1.693 \times 10^5$ N/m.

Neither in-plane frequencies nor modes change significantly from those presented in Ref. 2; i.e., $\beta_1 = \beta_2 = 90$ deg. Natural frequencies for out-of-plane modes, however, increase as the cables approach the vertical. The presence of torsion can be discerned as a discontinuous change of bending slope at joints between segments.

Essentially rigid-body rotational motion of the hoop occurs about an axis normal to its plane and through its center for small amplitudes, with cable stiffness $k_c = 1.693 \times 10^5$ N/m ($\gamma = 0$). But other primarily rigid-body motions, such as heaving, rocking, and in-plane translation, appear only when cable stiffness is decreased and/or when the stiffnesses of the hoop are increased significantly.

Cable stiffnesses which differ above and below the hoop destroy the symmetry of the configuration, and couple in-plane and out-of-plane motions.

Table 2 shows the coupled frequencies of the hoop for various cable angles and cable stiffnesses shown. Cable preload puts hoop segments in steady compression equal to 97.224 N. The two lowest frequency modes are primarily rigid motion for each cable angle.

Table 2 Coupled frequencies (rad/s)[a]

β, deg	γ	Coupled frequency	
$\beta_1 = 45$	0	$\omega_1 = 0.04$	$\omega_2 = 374.9$
$\beta_2 = 135$	$\pi/3$	$\omega_1 = 37.2$	$\omega_2 = 191.7$
	$2\pi/3$	$\omega_1 = 58.9$	$\omega_2 = 138.9$
	π	$\omega_1 = 86.4$	$\omega_2 = 112.62$
$\beta_1 = 60$	0	$\omega_1 = 0$	$\omega_2 = 206.4$
$\beta_2 = 120$	$\pi/3$	$\omega_1 = 35.9$	$\omega_2 = 177.56$
	$2\pi/3$	$\omega_1 = 63.78$	$\omega_2 = 127.9$
	π	$\omega_1 = 86.4$	$\omega_2 = 124.4$
$\beta_1 = 75$	0	$\omega_1 = 0$	$\omega_2 = 113.3$
$\beta_2 = 105$	$\pi/3$	$\omega_1 = 23$	$\omega_2 = 164.73$
	$2\pi/3$	$\omega_1 = 61.94$	$\omega_2 = 116.36$
	π	$\omega_1 = 86.4$	$\omega_2 = 128.81$

[a]$k_{c_1} = 1.693 \times 10^3$ N/m, $k_{c_2} = 1.693 \times 10^5$ N/m.

Figure 4 shows these rigid modes, and Fig. 5 the first three coupled modes for $\beta_1 = 45$ deg and $\beta_2 = 135$ deg. These complicated coupled modes are shown in three ways; in-plane and out-of-plane motions separately and the full simultaneous motions in one diagram.

Conclusions

Transfer matrix analysis and application of the laws of cyclic symmetry are again shown to provide a useful technique for the increasingly complex configurations representing polygonal hoop platforms supported by preloaded cables, in this instance when the cables are configured out of the hoop plane. Asymmetry of cables about the hoop plane, as would be expected, results in modes that are coupled as regards motions in and out of that plane. For certain hoop segment and cable properties and geometries, free vibratory modes exist that consist of mostly rigid-body hoop motion. The transfer matrix, cyclic symmetry approach predicts those modes accurately, as shown by comparisons with simplified solutions assuming perfect hoop rigidity.

References

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